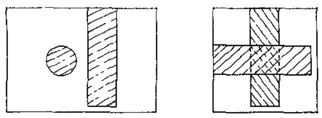

Fig. 1. Orthogonally coupled, capacitively loaded TEM resonators.

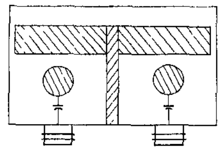

Fig. 2. UHF bandpass filter utilizing folded geometry.

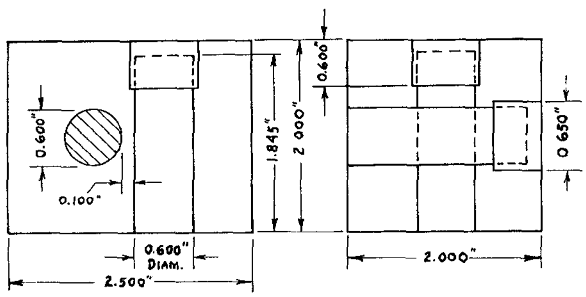

Fig. 3. Two-resonator filter with coaxial loading capacitors.

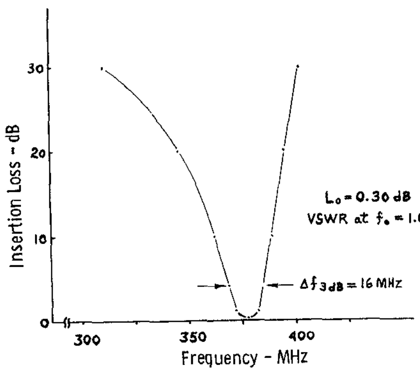

Fig. 4. Passband response of a two-resonator filter.

nal to its neighbor, the coupling between non-adjacent resonators predominated. A folded geometry, however, was found to provide a solution to this problem whereby orthogonal-resonator pairs are folded back as shown in Fig. 2.

A two-section filter with $\lambda_0/16$ capacitively loaded resonators was built using copper construction. The coaxial loading capacitors used a teflon dielectric and were mechanically adjustable. The filter configuration is shown in Fig. 3. The impedance of the resonators was 75 ohms in order to maximize the unloaded Q. The coupling was empirically adjusted to optimize the passband characteristic. The measured response of this two resonator filter is shown in Fig. 4. The overall unloaded Q was determined to be 1400, which means that a Q of 5000 was achieved for the coaxial loading capacitor. The total internal volume of the two resonator filter was 10 in^3. The performance is comparable to the best that can be achieved with an equivalent helical resonator filter.

Elio A. Mariani
U. S. Army Electronics Command
Electronic Components Lab.
Fort Monmouth, N. J.

Propagation in a Longitudinally Magnetized Ferrite-Filled Square Waveguide

Abstract—**The normal modes of a ferrite-filled longitudinally magnetized square waveguide are found using the generalized telegraphist's equations. The solutions are right- and left-hand circularly polarized waves. The predicted differential phase shift is within 10 percent of the experimental result.**

I. Introduction

Application of ferrite phase shifters in phased-array radars has generated a new interest in electromagnetic wave propagation through waveguides containing longitudinally magnetized ferrite. Exact solutions have been obtained only for the cylindrical waveguide symmetrically loaded with ferrite [1]–[3]. For the general case of ferrite-loaded rectangular waveguide, only approximate solutions have been reported [4]–[7]. In this correspondence, an approximate solution is given for the case of a fully filled square waveguide with longitudinal magnetization. The theoretical results are verified by the experimental measurement of phase shift in such a waveguide.

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II. Normal Modes

In order to obtain an approximate solution to the problem of a longitudinally magnetized ferrite in a square waveguide, the generalized telegraphist's equations as derived by Schelkunoff [8] are used. This method describes waveguide propagation in the ferrite-loaded guide in terms of coupling between the normal modes of the "uncoupled" waveguide.

Consider a ferrite-filled square waveguide of side a with propagation and applied magnetizing field in the $+z$ direction. The dominant modes of the empty waveguide are the degenerate pair, TE_{10} and TE_{01}. Schelkunoff couples these modes through the tensor permeability of the ferrite and obtains the following set of transmission line equations:

$$\frac{dV_{10}}{dz} = \mu_0\left(-j\omega\mu I_{10} + \omega\kappa\frac{8}{\pi^2}I_{01}\right) \tag{1}$$

$$\frac{dI_{10}}{dz} = -\left(j\omega\epsilon + \frac{\pi^2}{j\omega\epsilon a^2}\right)V_{10} \tag{2}$$

$$\frac{dV_{01}}{dz} = \mu_0\left(-\omega\kappa\frac{8}{\pi^2}I_{10} - j\omega\mu I_{01}\right) \tag{3}$$

$$\frac{dI_{01}}{dz} = -\left(j\omega\epsilon + \frac{\pi^2}{j\omega\epsilon a^2}\right)V_{01} \tag{4}$$

where μ and κ are elements of the Polder tensor [9] which at low magnetic fields may be written as

$$\mu \approx 1 \tag{5}$$

$$\kappa \approx -\frac{\omega_m}{\omega} \qquad \omega_m = 2\pi\gamma(4\pi M_s) \tag{6}$$

where

$4\pi M_s$ = ferrite saturation magnetization
γ = gyromagnetic ratio (2.8 MHz/Oe)
ω = microwave radian frequency.

The quantities V and I represent normalized mode voltages and currents. Proceeding as Schelkunoff, define

$$\beta_0 = \left[\omega^2\mu_0\epsilon - \left(\frac{\pi}{a}\right)^2\right]^{1/2} \tag{7}$$

$$k = -\frac{8}{\pi^2}\frac{\kappa}{\mu}. \tag{8}$$

Differentiation of (2) and (4) and substitution into (1) and (3) yields

$$\frac{1}{\beta_0^2}\frac{d^2I_{10}}{dz^2} = -\mu(I_{10} - jkI_{01}) \tag{9}$$

$$\frac{1}{\beta_0^2}\frac{d^2I_{01}}{dz^2} = -\mu(jkI_{10} + I_{01}). \tag{10}$$

Assuming solutions of the form

$$I_{10} = I_0e^{-j\beta z} \tag{11}$$

$$I_{01} = I_0'e^{-j\beta z} \tag{12}$$

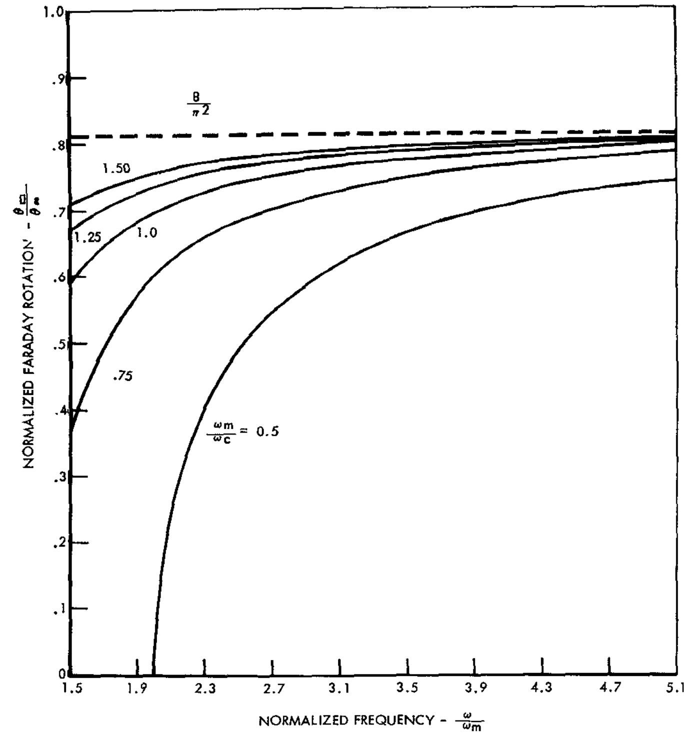

Fig. 1. Ratio of Faraday rotation in square waveguide to that in an infinite ferrite as a function of frequency. The ratio of ferrite saturation magnetization frequency to the cutoff frequency of the waveguide is the parameter.

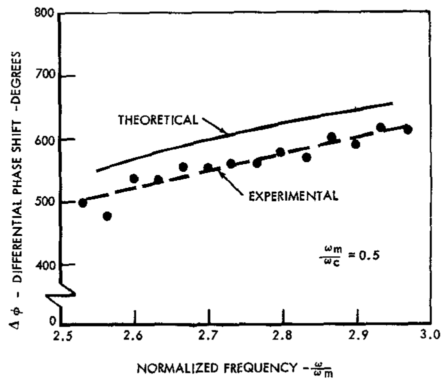

Fig. 2. Comparison of theoretical and experimental frequency-dependence of differential phase shift for square waveguide.

and substituting in (9) and (10) gives

$$I_0^2 = -I_0'^2$$

or

$$I_0 = \pm jI_0' \tag{13}$$

which is the condition for circular polarization. Thus the normal modes are right and left circularly polarized waves whose propagation constants are

$$\frac{\theta_\square}{\theta_\infty} = \frac{[1-(\omega_c/\omega)^2]^{1/2}[\sqrt{\mu+(8/\pi^2)\kappa}-\sqrt{\mu-(8/\pi^2)\kappa}]}{\sqrt{\mu+\kappa}-\sqrt{\mu-\kappa}} \tag{18}$$

$$\beta_+ = \beta_0\sqrt{\mu+\frac{8}{\pi^2}\kappa} \tag{14}$$

$$\beta_- = \beta_0\sqrt{\mu-\frac{8}{\pi^2}\kappa}. \tag{15}$$

Since the normal modes in an infinite ferrite medium are also circularly polarized [10], it is of interest to compare (14) and (15) with the propagation constants for the infinite medium:

$$\beta_+ = \omega\sqrt{\mu_0\epsilon}\sqrt{\mu+\kappa} \tag{16}$$

$$\beta_- = \omega\sqrt{\mu_0\epsilon}\sqrt{\mu-\kappa}. \tag{17}$$

The normalized Faraday rotation in the square waveguide is given by

where θ is the Faraday rotation and

$$\omega_c = \frac{\pi}{a\sqrt{\mu_0\epsilon}}. \tag{19}$$

The boundary conditions imposed by the waveguide system decrease the amount of Faraday rotation. The factor $[1-(\omega_c/\omega)^2]^{1/2}$ in (18) is the dispersion associated with the waveguide, and $8/\pi^2$ is a coupling factor which accounts for the transverse variation of the fields. Note in this approximation that, although the normal modes are circularly polarized waves, the fields are circularly polarized only along the diagonals of the waveguide.

Equation (18) is plotted in Fig. 1 as a function of frequency. For a fixed value of ω_m, the Faraday rotation increases as ω_c decreases since the dispersion is less. For large ω the curves asymptotically approach the value $8/\pi^2$ rather than unity since this analysis assumes only TE_{10} and TE_{01} modes in the waveguide.

A similar analysis in circular waveguide has been done by Severin [11] who found a maximum coupling factor of 0.83 ($\approx 8/\pi^2$). Thus, circular waveguide provides no advantage in terms of coupling coefficient over a square waveguide.

III. Experimental Results

Experimental verification of this theory may be obtained in either of two ways. The first is to excite the waveguide with a linearly polarized signal and to measure the Faraday rotation as this signal propagates down the guide. The Faraday rotation is

$$\theta = \frac{(\beta_+ - \beta_-)l}{2}. \tag{20}$$

The second method is to excite the waveguide with either sense of circular polarization, i.e., one of the normal modes, and to measure the differential phase shift between positive and negative saturation of the ferrite. This phase shift is given by

$$\Delta\phi = (\beta_+ - \beta_-)l. \tag{21}$$

This was the method chosen for the measurement.

The test section was placed in one arm of a waveguide bridge. With the ferrite latched to the remanent flux in one direction, a reference phase was measured. The flux was then reversed, and the differential phase shift was measured as a function of frequency; the results are plotted in Fig. 2. The theoretical curve was calculated using published values for the remanent magnetization of the ferrite. From the excellent agreement obtained, it may be concluded that the normal modes of the longitudinally magnetized square waveguide are right and left hand circularly polarized waves.

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W. E. Hord
Emerson Electric Company
St. Louis, Mo.

F. J. Rosenbaum[1]
Dept. of Elec. Engrg.
Washington University
St. Louis, Mo.

[1] Also Consultant to Emerson Electric Company.

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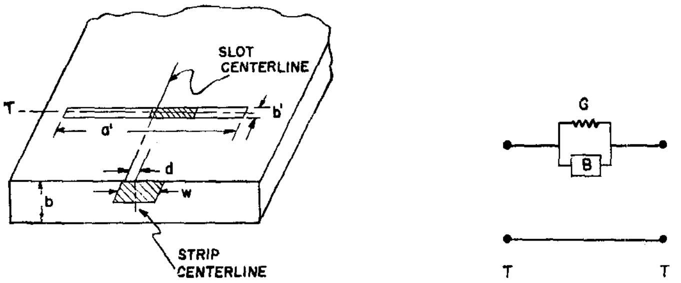
Conductance Data for Offset Series Slots in Stripline

***Abstract*—Frequency-independent curves are presented which can be used to calculate the conductance of offset series slots in stripline for 50-ohm characteristic impedance and three common ground plane spacings. Experimental verification is given for a particular case.**

There appear to be no data presently available for the conductance of offset series slots in stripline. This would be particularly useful in the design of stripline antenna slot arrays. The general stripline and slot configuration plus equivalent circuit are shown in Fig. 1. A very simple modification is made to Oliner's[1] expression for the conductance of a narrow centered series slot in an infinite perfectly conducting ground plane, which from his paper is

$$\frac{G}{Y_0} = \frac{16}{3\pi}\frac{K(k')}{K(k)}\left(\frac{a'}{\lambda}\right)^2\left[1 - 0.374\left(\frac{a'}{\lambda}\right)^2 + 0.130\left(\frac{a'}{\lambda}\right)^4\right] \quad (1)$$

where



(a) (b)

Fig. 1. (a) Offset series slot configuration. (b) Equivalent circuit.

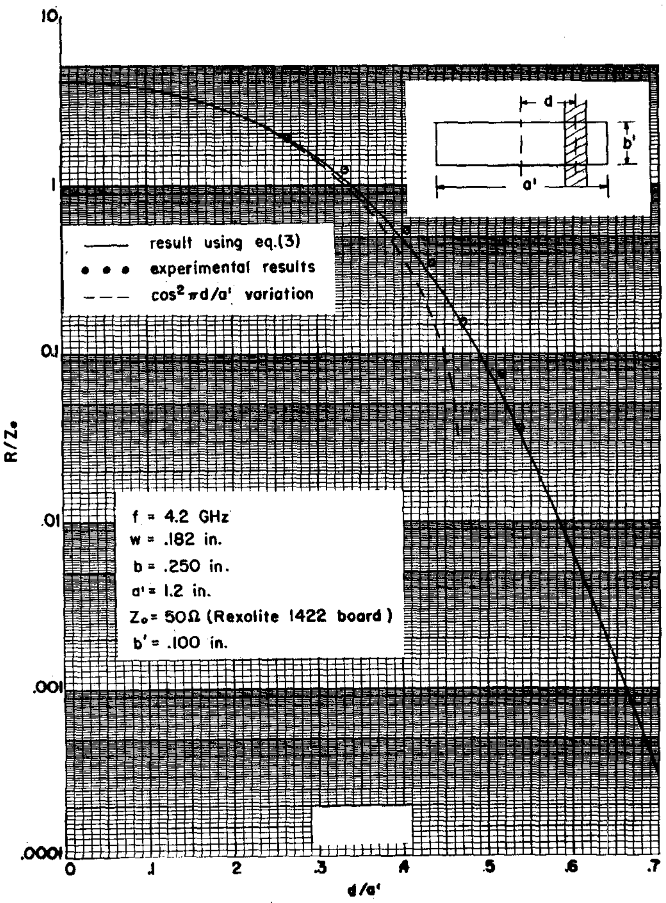


Fig. 2. Variation of resonant R/Z_0 with series slot offset in stripline.

$k = \tanh \pi w/2b$
w = strip width
b = ground plane spacing
$k' = +(1-k^2)^{1/2}$
a' = slot length
b' = slot width
d = slot offset, and is the distance from the slot centerline to the strip centerline
$K(k)$ = complete elliptic integral of the first kind.

The denominator of the G/Y_0 expression originally contained the integral

$$I^2 = \left[\int_{(-a'/2)-d}^{(a'/2)-d} \frac{\cos[\pi(x+d)/a']dx}{[1 + k'^2 \sinh^2(\pi x/b)]^{1/2}}\right]^2 \quad (2)$$

where $d=0$ was used in Oliner's case. Equation (2) simply contains an offset cosine assumed for the electric field distribution in the slot offset by d from the strip centerline ($x=0$). One could assume the cosine in (2) to be slowly varying with respect to the denominator at $x=0$ for small offset, say for $d/a' < 0.3$. This would simply give Oliner's result in (1), multiplied by $\sec^2(\pi d/a')$. For a slotted stripline array, however, the useful range is for $d/a' > 0.3$, and a numerical integration of (2) as it stands seems most convenient.

If I_0 is the value of (2) for $d=0$, the expression for G/Y_0, modified to include slot offset

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